Closing Wed: HW_1A, 1B
Closing Fri: HW_1C
Check out the first newsletter!

4.9 Antiderivatives (continued)

Example:

$$f''(x) = \frac{3}{\sqrt{x}}$$

$$f(1) = 0, f(4) = 1$$

Find $f(x)$.

Example:

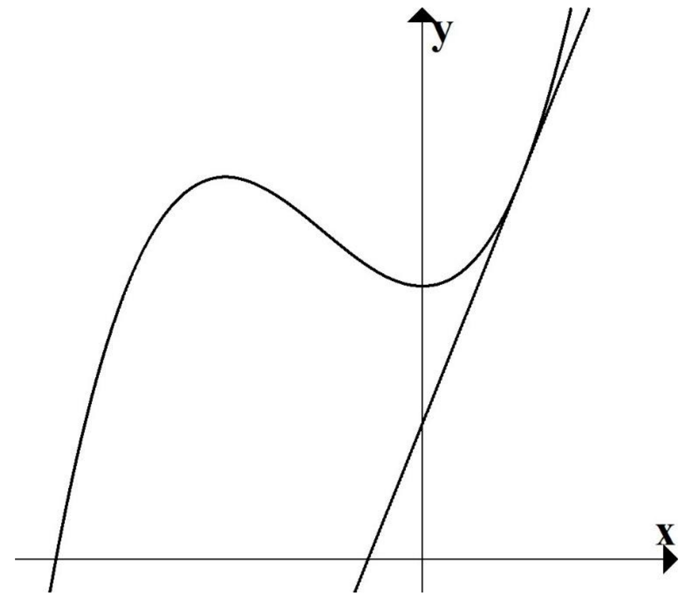
Ron *steps off* the 10 meter high dive at his local pool. Find a formula for his height above the water.

(Assume his acceleration is a constant 9.8 m/s^2 downward)

5.1 Defining Area (Riemann sums)

Calculus is based on limiting processes that "approach" the exact answer to a rate question.

Calc. I Visual:

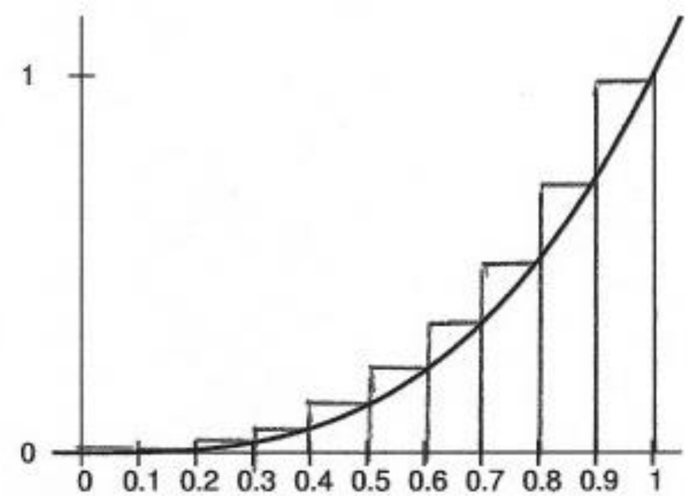


In Calculus I, you defined

f'(x) = `slope of the tangent at *x*'

$$= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Calc. II Visual:



$R_{10} = 0.3025$

In Calculus II, we will see that antiderivatives are related to the area `under' a graph

$$= \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i{}^*)\Delta x$$

Riemann sums set up:

We build a procedure to get better and better approximations of the area “under” $f(x)$.

1. Break into *n* subintervals.

   $\Delta x = \frac{b-a}{n}$ and $x_i = a + i\Delta x$

2. Draw *n* rectangles.

   Area of each rectangle =

   (height)(width) $= f(x_i^*)\Delta x$

3. Add up rectangle areas.

Example:
Approx. the area under $f(x) = x^3$ from x = 0 to x = 1 using n = 3 subdivisions and *right-endpoints* to find the heights.

You do:

Approx. the area under $f(x) = x^3$ from x = 0 to x = 1 using n = 4 subdivisions and *left-endpoints* to find the heights.

I did this again with 100 subdivisions, then 1000, then 10000.

Here is a summary of my findings:

n	R_n	L_n
4	0.390625	0.140625
5	0.36	0.16
10	0.3025	0.2025
100	0.255025	0.245025
1000	0.25050025	0.24950025
10000	0.2499500025	0.2500500025

General Pattern: (right-endpoint)

For $f(x) = x^3$ on x = 0 to x = 1.

$$\Delta x = \frac{1-0}{n} = \frac{1}{n}$$

$$x_i = 0 + i\frac{1}{n} = \frac{i}{n}$$

$$\text{Height} = f(x_i) = x_i^3 = \left(\frac{i}{n}\right)^3$$

$$\text{Area} = f(x_i)\Delta x = x_i^3\Delta x = \left(\frac{i}{n}\right)^3\frac{1}{n}$$

Adding up the rectangle areas

$$\text{Sum} = \sum_{i=1}^{n} x_i^3\Delta x = \sum_{i=1}^{n}\left(\frac{i}{n}\right)^3\frac{1}{n}$$

$$\text{Exact Area} = \lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{i}{n}\right)^3\frac{1}{n}$$

Example: Approximate the area under $f(x) = 1 + x^2$ from $x = 2$ to $x = 3$ using $n = 4$ Riemann sums and right endpoints.

What is the general pattern in terms of *n*?

$\Delta x =$

$x_i =$

$$\sum_{i=1}^{n} f(x_i)\Delta x =$$

Another Example:

Using sigma notation, write down the general Riemann sum definition of the area from x = 5 to x = 7 under

$$f(x) = 3x + \sqrt{x}$$

$$\Delta x = \frac{b-a}{n} =$$

$$x_i = a + i\,\Delta x$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x =$$

Velocity/Distance & Reimann Sums

When velocity is a ***constant***:

Distance = Velocity · Time

Example:

You are accelerating in a car. You get the following measurements:

t (sec)	0	0.5	1.0	1.5	2.0
v(t) (ft/s)	0	6.2	10.8	14.9	18.1

Estimate the distance traveled by the car traveled from 0 to 2 seconds.

5.2 The Definite Integral

Def'n:

We define the **definite integral of f(x) from x = a to x = b** by

$$\int_a^b f(x)dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x\,,$$

where $\Delta x = \frac{b-a}{n}$ and $x_i = a + i\Delta x$.

Basic Integral Rules:

1. $\int_a^b c\, dx = (b-a)c$

2. $\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$

3. $\int_a^b cf(x)\, dx = c\int_a^b f(x)dx$

and

$\int_a^b f(x) + g(x)\, dx$

$= \int_a^b f(x)dx + \int_a^b g(x)\, dx$

4. $\int_b^a f(x)dx = -\int_a^b f(x)dx$

Examples:

1. $\int_4^{10} 5\, dx =$

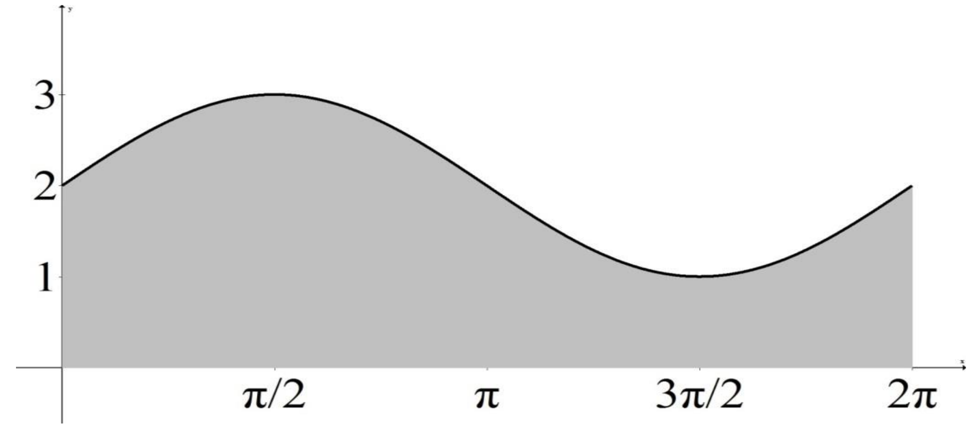
2. $\int_0^3 x^2 dx + \int_3^7 x^2 dx =$

3. $\int_0^4 5x + 3\, dx =$

4. $\int_3^1 x^3 dx = -\int_1^3 x^3\, dx$

Note on quick bounds (HW_1C: 9,10)

$$m(b-a) \leq \int_a^b f(x)dx \leq M(b-a)$$



Example: Consider the area under

f(x) = sin(x) + 2

on the interval *x = 0* to *x = 2π.*

(a) What is the max of f(x)? (label M)

(b) What is the min of f(x)? (label m)

(c) Draw **one** rectangle that contains all the shaded area? What can you conclude?

(d) Draw **one** rectangle that is completely inside the shaded area? Conclusion?